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THE DETERMINATION OF ELASTIC CONSTANTS OF NEMATIC LIQUID CRYSTALS FROM NOISE MEASUREMENTS OF SCATTERED LASER LIGHT

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A new optical technique is introduced for determining the elastic constants of nematic liquid crystals by measuring the spectral noise intensity of scattered laser light.

I. INTRODUCTION

In a nematic liquid crystal three types of distortion can be distinguished, each of which is characterized by a specific elastic constant.[1] These static quantities can be determined via light scattering experiments by measuring the angular dependence of the total cross section.[2] However, experimental problems will arise because of refraction phenomena at the walls of the sample. These problems are caused mainly by poorly defined angular dependent reflection, interference phenomena and a change in the subtended solid angle.[3] In addition the measurements may be impaired by the occurrence of stray light.

In a liquid crystal the elongated molecules fluctuate around a mean direction called the director. It is because of these orientation fluctuations that light is strongly scattered. Relaxation effects that are coupled with the three types of distortion give rise to Lorentzian line broadening of the scattered light, from which a relaxation time can be determined. This relaxation time provides us with direct information about the dynamics of the nematics, namely the distortion viscosity constants.[4]

The experimental problems that one normally associates with total cross section measurements are of little consequence in measurements of the spectral linewidth of the scattered light. Because of the three types of distortion one generally measures a spectrum of a sum of Lorentzian broadened lines each with an intensity that is weighted by functions depending on optical parameters and on the elastic constants.

The linewidth as determined from noise data has been published earlier.[5,6]

As this paper will show we can obtain the three bandwidths separately and the ratio of the elastic constants by making a computer analysis of noise data obtained as a function of the scattering angle.

We shall demonstrate this for the case of an optical configuration in which only the splay and twist deformation phenomena are observed.

II. THEORY

The expression for the noise intensity spectrum of scattered laser light is given by:[5,6]

$$S_{\Delta I}(\omega) = M^2 \left\{ \sum_{\alpha=1,2} \Gamma_\alpha^{\,2} \frac{2u_{s_\alpha}}{[\omega^2+(2u_{s_\alpha})^2]} + 2\Gamma_1\Gamma_2 \frac{(u_{s_1}+u_{s_2})}{[\omega^2+(u_{s_1}+u_{s_2})^2]} \right\}, \quad (1)$$

where $\Gamma_\alpha \equiv G_\alpha^{\,2}/\kappa_\alpha(\vec{q})$, with $G_\alpha \equiv (f_\alpha i_3+f_3 i_\alpha)$; M is a constant. The vectors $\vec{i}$ and $\vec{f}$ are the polarization vectors of the incoming and scattered light respectively. The vector components are considered with respect to an orthonormal coordinate system defined by the following base vectors $\hat{e}_3 \equiv \vec{n}$, $\hat{e}_2 \equiv (\vec{n}_0\times\vec{q})/|\vec{n}_0\times\vec{q}|$, $\hat{e}_1 \equiv \hat{e}_2\times\hat{e}_3$, where $\vec{q}$ is the scattering wave vector and $\vec{n}_0$ represents the director. The function $\kappa_\alpha(\vec{q})$ is equal to $K_\alpha q_\perp^{\,2}+K_3 q_{/\!/}^{\,2}$ (α = 1, 2), where K_1, K_2 and K_3 represent the splay, twist and bend elastic constants respectively.[2] The components $q_{/\!/}$ and $q_\perp$ of the scattering wave vector are parallel and normal to the director. In our case we took the polarization of the incoming light to be $\vec{i}$ = [0 1 0]; therefore $q_{/\!/} = 0$ and $\kappa_\alpha = K_\alpha q_\perp^{\,2}$. Hence denoting splay and twist by α = 1, 2 respectively we observe that the spectral intensity $S_{\Delta I}$ can be considered to be a sum of three Lorentzians, associated with *pure* splay, *pure* twist and a splay/twist cross term, the half-bandwidths of which are related to the viscoelastic properties,[5] i.e. $(\omega_{\frac{1}{2}})_\alpha = 2u_{s\alpha} = 2\kappa_\alpha/\eta_\alpha$. In our configuration it holds that $u_{s\alpha} = (K_\alpha/\eta_\alpha) q_\perp^{\,2}$. Although the measured noise spectrum itself is not a pure Lorentzian, but is a sum of three Lorentzians, it is still possible to determine its half-width.

This result depends on the well-known optical parameters G_α and $q_\perp$ as well as on three unknown ratios, i.e. η_1/K_1, η_2/K_2 and K_1/K_2, which are constants (see eq. (1)).

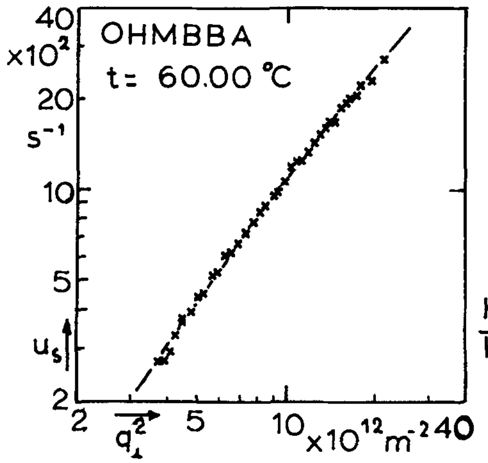

FIGURE 1 An example of u_s versus $q_\perp^2$. The crosses represent the experimental data; the straight curve is the best fit with $\eta_1/K_1 = 7.2 \cdot 10^9$ sm^{-2}, $\eta_2/K_2 = 1.45 \cdot 10^{10}$ sm^{-2} and $K_1/K_2 = 1.8$.

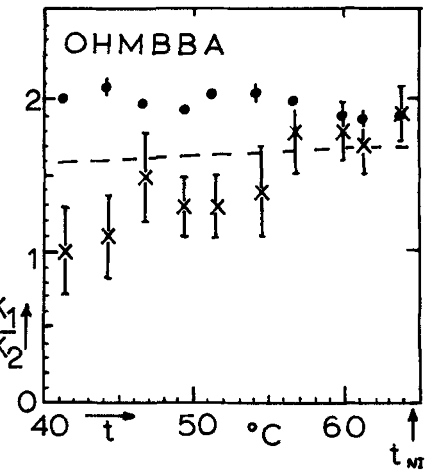

FIGURE 2 Results of K_1/K_2 versus temperature. The crosses are from the fitting procedure; the dots are from light intensity distributions[7]; the dashed curve is the best fit for data obtained from Freedericksz transitions.[8]

By measuring the noise spectrum at several different scattering angles, the half-bandwidths can be found as a function of G_α and $q_\perp$, because the latter parameters are functions of the scattering angle. With the help of a computer one can obtain a best fit to these data by using the following ratios as fitting parameters: η_1/K_1, η_2/K_2 and K_1/K_2. Therefore not only one can obtain the viscoelastic ratios from noise measurements[5] but one can also find the ratio of the elastic constants.

III. EXPERIMENTAL RESULTS

In figure 1 results are shown for OHMBBA (O-hydroxy-p-methoxybenzylidene-p'-butylaniline) at a temperature $t = 60.00°C$. In the figure the experimental data for $u_s \equiv \frac{1}{2}(\omega_{\frac{1}{2}})_{exp.}$ are plotted on a log-log scale versus $q_\perp^2$. The solid line is the best fit to the experimental data. This best fit is obtained with the help of a least squares method on a log-log scale. The fitted values for the viscoelastic and elastic ratios are indicated in figure 1. Figure 2 shows results for K_1/K_2 at different temperatures. In addiation we present the results obtained from measurements of scattered light intensity distributions[7] and from measurements of Freedericksz transitions.[8]

IV. CONCLUSION

It has been shown that spectral noise intensity measurements not only give us information about dynamical quantities but they also tell us about static quantities such as the splay/twist elastic ratio, K_1/K_2. The error in K_1/K_2 is mainly caused by experimental errors in $(\omega_{\frac{1}{2}})_{exp.}$. As is shown in figure 2 this statistical error is rather large compared to the statistical errors associated with the other two methods. It should be noted, however, that the different methods yield results which differ by more than can be accounted for by statistical errors. Apparently the errors involved are of a systematic nature.

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